Appendix B: A comment on multi-species perturbations

In previous qualitative modelling studies, the response of a species to a multi-species press-perturbation was modelled by summing the corresponding elements in the sensitivity matrix (e.g. Raymond et al. (2011)). However, we note that summing elements of the sensitivity matrix in this way implicitly assumes that the magnitude of the perturbation on each species is equal. In the context of pest management, these perturbations represent the outcomes of some future pest-control program. Given that the outcome of a future program cannot be known, we advise against this practice, and we do not use it in our study.

In the case of a single-species perturbation, the effect on species i of a press-perturbation of species j is found by rearranging Equation 12 of Nakajima (1992):

$$d x_i = s_{i,j} d z_j \tag{1}$$

where $d x_i$ is the change in the steady-state population size of species i or the focal species' response, $d z_j$ is the rate in inflow or removal of species j, and $s_{i,j}$ is the corresponding element of the sensitivity matrix. In this case, the sign of the species response $d x_i$ in Equation 1 can be determined without knowing the magnitude of the perturbation (assumed small).

In the case of a multi-species perturbation, the effect on species i of simultaneous press-perturbation of a set of species $j \in P$ is found similar to Equation 15 in Nakajima (1992):

$$d x_i = \sum_{j \in P} s_{i,j} d z_j \tag{2}$$

In this case, if there is a mix of signs of elements $s_{i,j}$ in the sum in Equation 2, then the sign of the species response $d x_i$ cannot be determined without knowing the relative magnitudes of the perturbations $d z_j$.

When Raymond et al. (2011) summed the elements of the sensitivity matrix to obtain the sign of the species' responses to a multi-species press-perturbation, they were effectively assuming that all $d z_j$ in

Equation 2 were equal in magnitude. However it is almost certain that the rates of pest kills was not equal — not in their application, nor in ours, nor in general. Therefore we did not take this approach and we generally advise against it.

To evaluate Equation 2 correctly, one must know the true values of $d\,z_j$. One would need to know both the size of the steady-state pest populations currently and the rate of removal of each pest species from the future pest control program, or equivalently, the relatively proportional rates of pest-species removal that will occur in the future pest control program. We did not have this information in our study.

Reference

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